THE MATHEMATICS BEHIND ROCKET LEAGUE

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Introduction

As a teenage boy I have played a lot of videogames, however quite recently I have been playing a game called Rocket League. Rocket league is currently one of the biggest games in the world, and the premise behind it very simple. It is basically a game of football however you don't play with humans you play as a car that has the ability to jump and boost like a rocket hence the name "Rocket League. As you can imagine there is a lot of freedom for the car to move and along the field and even in the air due to the rocket boost, therefore there are a numerous number of special moves a player can make to get the upper hand on their opponents ranging in multiple difficulties.

Recently I had entered a tournament that – I had not realized until I played my first game – was above my skill level, meaning that the other contestants in the tournament were in a much higher skill level than I am. After the first game I had lost by a large difference, in fact I hadn't even scored a goal, and by that point I knew I was outmatched by everyone else in the roster. Unfortunately, I was kicked out after three games in the grouping stage, which was expected but still disappointing. I immediately started to analyze the games that I had played in the tournament to get some ideas on how to improve my skills.

When I finished watching the games, I had noticed that in all of my matches I was struggling quite a lot with intercepting the ball midair as I was missing it quite regularly which meant my opponent could gain possession very quickly. This was very bad for me as my opponent was able to score easily after this as there was a window in which I was unable to get the ball because I was midair and he had time to score. Since the ball was just an ordinary ball it obviously followed projectile motion meaning I am able to model the path of the ball. This means that with the use of mathematics I am able to accurately predict and hit the ball midair.

Aims

In this exploration I will be mathematically modelling; the motion of the ball and the car before the first collision and, the motion of the ball after the collision. The aim of the exploration is use the models in order to find the range of angles that the car needs to tilt in order to hit the ball directly into the goal.

Objectives

The steps involved in the exploration are:

1. Modelling the motion of the ball and the car before collision.
2. Finding the point of collision using given ranges of angles and velocities of the ball.

3. Modelling the motion of the ball after the collision.

4. Finding the ranges of angles at which the car needs to tilt in order to hit the ball directly into the goal using the above.

Building cases

With most videogames it has always been quite difficult to gather information about the inner workings of the game such as the mechanics. This game was no different as the official game doesn't post such information instead I had to use a chat log with an official game developer to find out the relevant information such as the car's maximum velocity and the dimensions of the court. Consider a vehicle, with the ability to jump and boost in the air and a ball, on a $300.1m \times 375.5m \times 100m$ pitch. The acceleration due to gravity in this world is less than the one on our earth and this is favorable as it provides more airtime for the game's true mechanics to be shown. The car has a maximum velocity of $51.85ms^{-1}$. In the game the acceleration due to gravity is $g = 6.5ms^{-2}$. Below shows a birds eye view of the pitch in Rocket League:

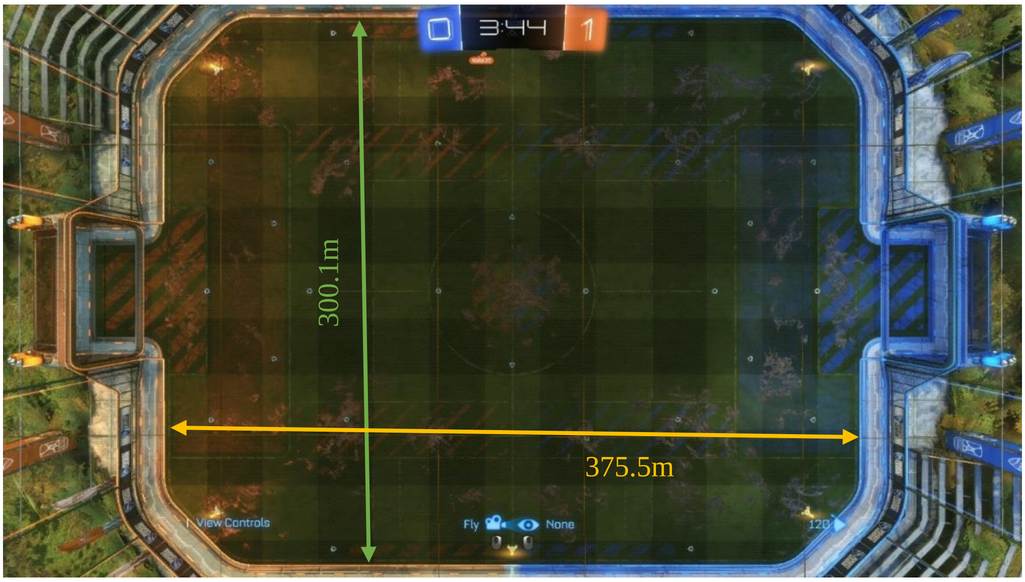
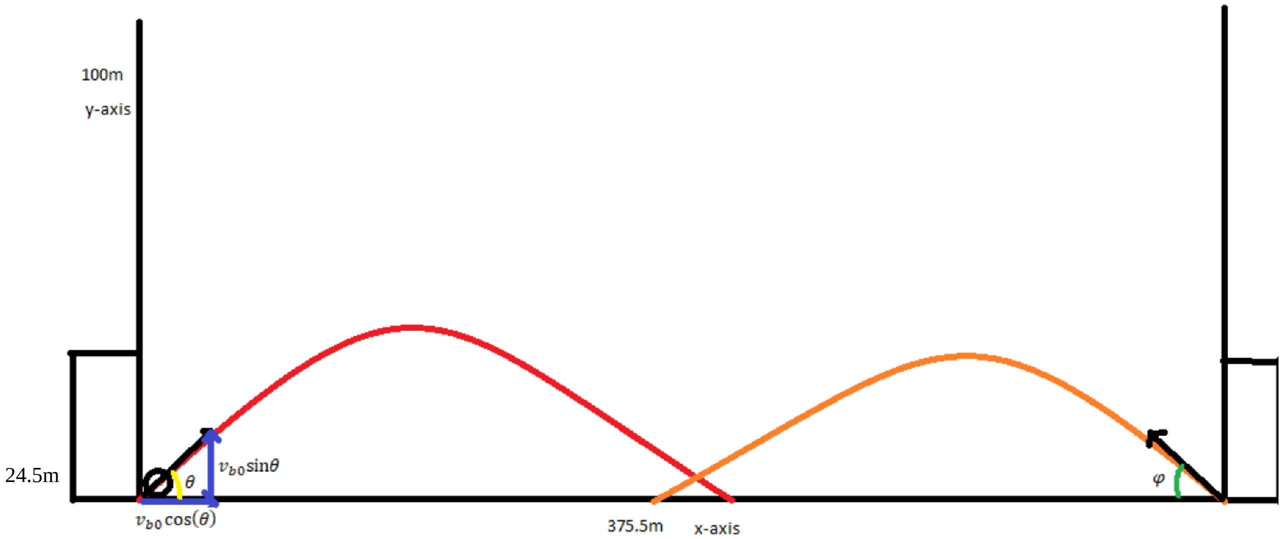


Figure 1 Shows a birds eye view of the pitch in rocket league

On one end of the pitch a ball is projected at an angle θ with an initial velocity, v_{b0}. The range of angles the ball is being projected will be $20° \leq \theta \leq 70°$ and the initial velocity varies from $30ms^{-1} \leq v_{b0} \leq 70ms^{-1}$ because from my extensive gameplay I had found that these seemed to be the most common values. I will take the point at which the ball is initially placed to be the origin, which in this case is on the left goal line. Let $v_{bx}\ and\ v_{by}$ be the horizontal and vertical velocities of the ball at any time t. Let $x_b\ and\ y_b$ be the horizontal and vertical displacements of the ball respectively at time t.

Assuming the car starts on the opposite goal line. Let the car be projected at an angle φ with the horizontal, with initial velocity of $51.85ms^{-1}$. Let x_c and y_c be the horizontal and vertical displacements of the car respectively at time t. The height of the stadium is 100m where there is a ceiling and the goal is at a height of 24.5m. The car and ball are projected at the same time. The diagram below depicts the setting of the pitch when time is 0.



In this exploration angles measured from the horizontal are considered as positive if it is measured in the anti-clockwise direction and as negative if measured in the clockwise direction.

Modelling the equation of motion of the ball

The vertical and horizontal components of the initial velocity are $v_{b0}\sin(\theta)$ and $v_{b0}\cos(\theta)$ respectively as shown in the diagram above.

To find v_{bx} and x_b:

Since there is no acceleration in the horizontal direction we can take:

$$\frac{d(v_{bx})}{dt} = 0$$

$$v_{bx} = \int 0\,dt$$

$$v_{bx} = c \text{ } where \text{ } c \text{ } is \text{ } a \text{ } consant$$

$$\therefore when \text{ } t = 0, v_{bx} = v_{b0}\cos(\theta)$$

$$\therefore v_{bx} = v_{b0}\cos(\theta) \text{ } for \text{ } all \text{ } t$$

Now that we have found the horizontal velocity, we will need to further integrate to find the horizontal displacement of the ball (x_b) at any given time t.

To find the x-coordinate:

$$x_b = \int v_{bx}\, dt$$

$$x_b = \int v_{b0} \cos(\theta)\ dt$$

$$x_b = v_{b0} \cos(\theta) t + c$$

$$x_b = 0\ when\ t = 0\ implies\ that\ c = 0$$

$$\therefore x_b = v_{b0} \cos(\theta)\, t$$

To find v_{by} *and* y_b:

Since the vertical motion of the ball is affected by acceleration due to gravity 'g'

$$\frac{d(v_{by})}{dt} = -g$$

$$v_{by} = -\int g\, dt$$

$$v_{by} = -gt + c$$

$$when\ t = 0, v_{by} = v_{b0} \sin(\theta)$$

$$\therefore v_{by} = v_{bo} \sin(\theta) - gt$$

We know that,

$$y_b = \int v_{by}\ dt$$

$$y_b = \int (v_{b0} \sin(\theta) - gt)\, dt$$

$$y_b = v_{b0} \sin(\theta)\, t - \frac{1}{2} gt^2 + c$$

$$\because the\ ball\ is\ projected\ from\ the\ origin, y_b = 0\ when\ t = 0\ it\ implies\ that\ c = 0$$

$$\therefore y_b = v_{b0}\sin(\theta)\,t - \frac{1}{2}6.5t^2; since\ g = 6.5ms^{-2}$$

$$y_b = v_{b0}\sin(\theta)\,t - 3.25t^2$$

We know that:

$$x_b = v_{b0}\cos(\theta)\,t \;\Rightarrow t = \frac{x_b}{v_{bo}\cos(\theta)}$$

$$substituting\ this\ expression\ for\ t, y_b = v_{b0}\sin(\theta)\left[\frac{x_b}{v_{b0}\cos(\theta)}\right] - 3.25\left[\frac{x_b}{v_{b0}\cos(\theta)}\right]^2$$

$$y_b = \tan(\theta)x_b - \frac{3.25{x_b}^2}{[v_{b0}\cos(\theta)]^2}$$

The above equation is the equation of motion for the ball's path which is a quadratic in x_b. Since the coefficient of ${x_b}^2$ is negative the graph of this function is a downward parabola.

Modelling the motion of the car:

The car is projected with an initial speed of $51.85ms^{-1}$and an angle of φ with the horizontal. The initial position of the car is (375.5, 0). Since the motion of the car is in the opposite direction of the ball's motion the horizontal velocity component will be negative. Therefore, we can get the following equations:

Horizontal Displacement of the car:

$$x_c = -51.85\cos(\varphi)\,t + 375.5$$

Vertical Displacement of the car:

$$y_c = 51.85\sin(\varphi)\,t - 3.25t^2$$

Car's equation of motion can be found by eliminating t between the above two equations:

$$y_c = \tan(\varphi)\,(375.5 - x_c) - \frac{3.25(375.5 - x_c)^2}{(51.85\cos(\varphi))^2}$$

Since the coefficient of ${x_c}^2$ is negative the graph of this function will also be a downward parabola.

Finding the point of collision

Notice how for now we don't have an angle for projection for the car this is because we still need to find the correct angle at which the car can be projected and actually hit the ball without missing it due to time.

At the point of intersection the horizontal displacements are equal.

$$x_b = x_c$$

$$v_{b0}\cos(\theta)\,t = -51.85\cos(\varphi)\,t + 375.5$$

$$v_{b0}\cos(\theta)\,t + 51.85\cos(\varphi)\,t = 375.5$$

$$t = \frac{375.5}{v_{b0}\cos(\theta) + 51.85\cos(\varphi)} \longrightarrow \text{Formula 1}$$

At the point of intersection the vertical displacements are equal.

$$\therefore y_b = y_c$$

$$v_{bo}\sin(\theta)t - 3.25t^2 = 51.85\sin(\varphi)\,t - 3.25t^2$$

$$v_{b0}\sin(\theta) = 51.85\sin(\varphi)$$

$$\varphi = \arcsin\left(\frac{v_{b0}\,sin(\theta)}{51.85}\right)$$

The formula above is defined when $0 \leq \frac{v_{b0}\sin(\theta)}{51.85} \leq 1$ since $v_{b0} > 0$ and $20° \leq \theta \leq 70°$.

Therefore $0 \leq v_{b0}\sin(\theta) \leq 51.85$. Also, we have assumed that $30ms^{-1} \leq v_{b0} \leq 70ms^{-1}$. By equating the two vertical displacement equations y_b *and* y_c, t got eliminated to get the value of φ. From this we can see that when the car is projected at an angle φ, given by the above formula, the vertical displacements of the car and ball are equal at any time t. Therefore the car and ball collide at t seconds given by Formula 1. This means that height can be used as an indicator as to whether the car is going to intercept the ball midair or not.

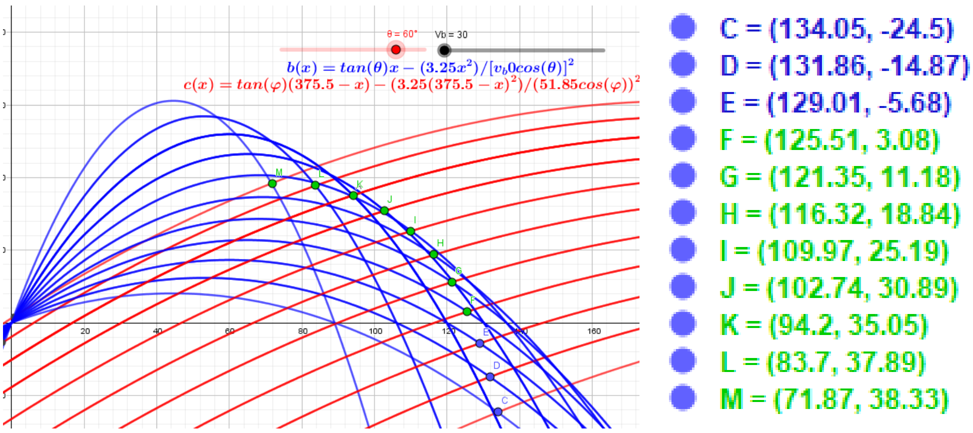
Using the range of values for θ and v_{b0} stated above, we can find the possible values of φ. I used Microsoft excel to generate the table storing the values of φ. The excel formula used in the below table was =DEGREES(ASIN((B$3*SIN($A4*PI ()/180))/51.85)). The values within the darker blue boxes are angles that won't project the car to intersect with the ball within the region of the court. The cells that are yellow are angles that project the car in a way that will allow them to intersect within the region of the court however, they are obstructed by the ceiling before doing so. Finally, the cells that are green contain angles that will project the car and successfully intersect the ball within the region of the court with no obstructions. I used the graphs of the functions representing the equation motion of the ball and car to determine which color the values should be in.

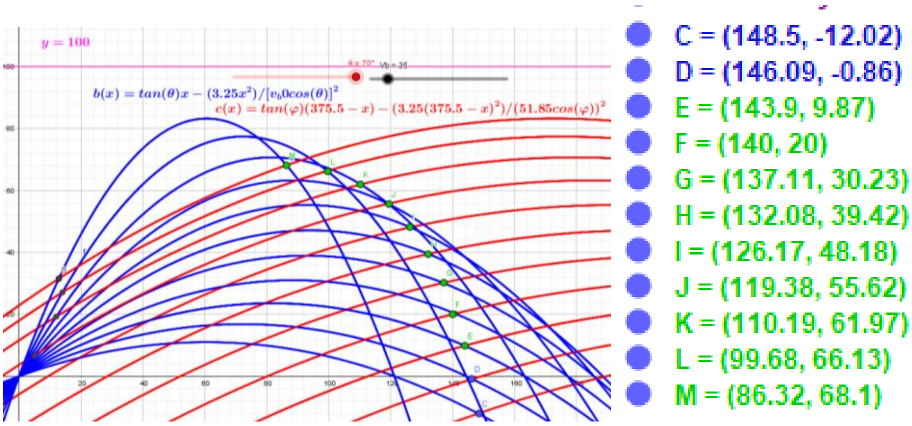
Angle at which the car is projected(ϕ°)									
Angle(θ°)	Velocity of the ball(m/s)								
	30	35	40	45	50	55	60	65	70
20	11.4	13.3	15.3	17.3	19.3	21.3	23.3	25.4	27.5
25	14.2	16.6	19.0	21.5	24.1	26.6	29.3	32.0	34.8
30	16.8	19.7	22.7	25.7	28.8	32.0	35.4	38.8	42.5
35	19.4	22.8	26.3	29.9	33.6	37.5	41.6	46.0	50.7
40	21.8	25.7	29.7	33.9	38.3	43.0	48.1	53.7	60.2
45	24.1	28.5	33.1	37.9	43.0	48.6	54.9	62.4	72.7
50	26.3	31.1	36.2	41.7	47.6	54.3	62.4	73.8	#NUM!
55	28.3	33.6	39.2	45.3	52.2	60.3	71.4	#NUM!	#NUM!
60	30.1	35.8	41.9	48.7	56.6	66.7	#NUM!	#NUM!	#NUM!
65	31.6	37.7	44.4	51.9	60.9	74.0	#NUM!	#NUM!	#NUM!
70	32.9	39.4	46.5	54.6	65.0	85.4	#NUM!	#NUM!	#NUM!
(green)	Intersecting								
(grey-blue)	Not Intersecting								
(yellow)	Obstructed								

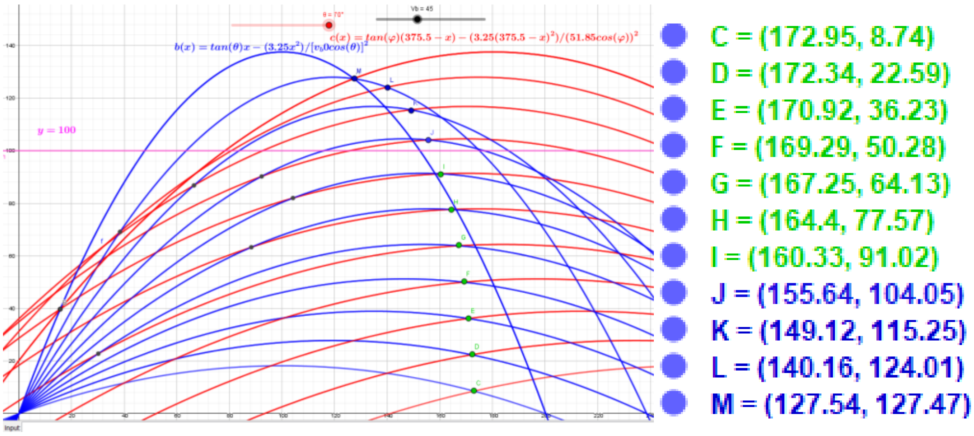
Table 1

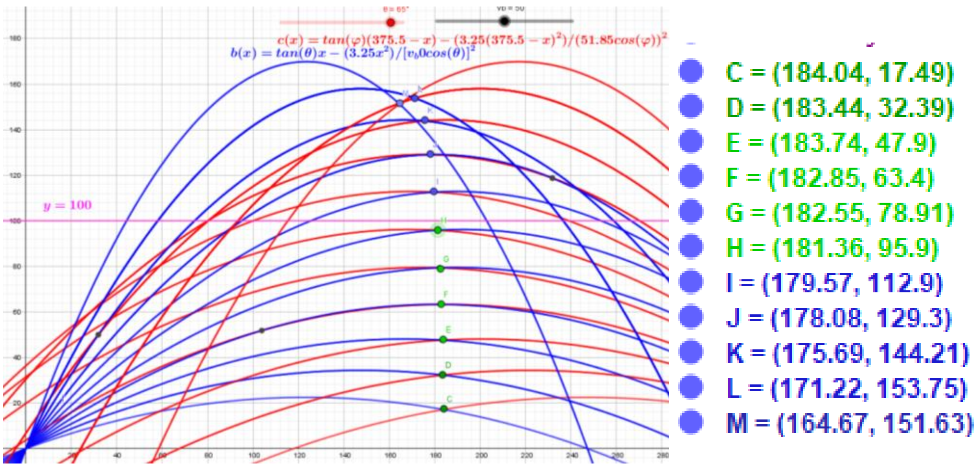
The table above will allow me to predict if I am able to hit the ball without any interference such as the ceiling or if the ball will even reach me. There are a few cells that contain "#NUM!" this is because the equation $\varphi = \arcsin\left(\frac{v_{b0}\,sin(\theta)}{51.85}\right)$ is defined when $0 \le v_{b0}\sin(\theta) \le 51.85$ and in those cells the value for $v_{b0}\sin(\theta)$ is greater than 51.85. The GeoGebra graphs are given below including, the line y=100 for the ceiling. I used the slider tool with the trace function to find whether the collision happens within the region of the court for various values of θ *and* v_b.

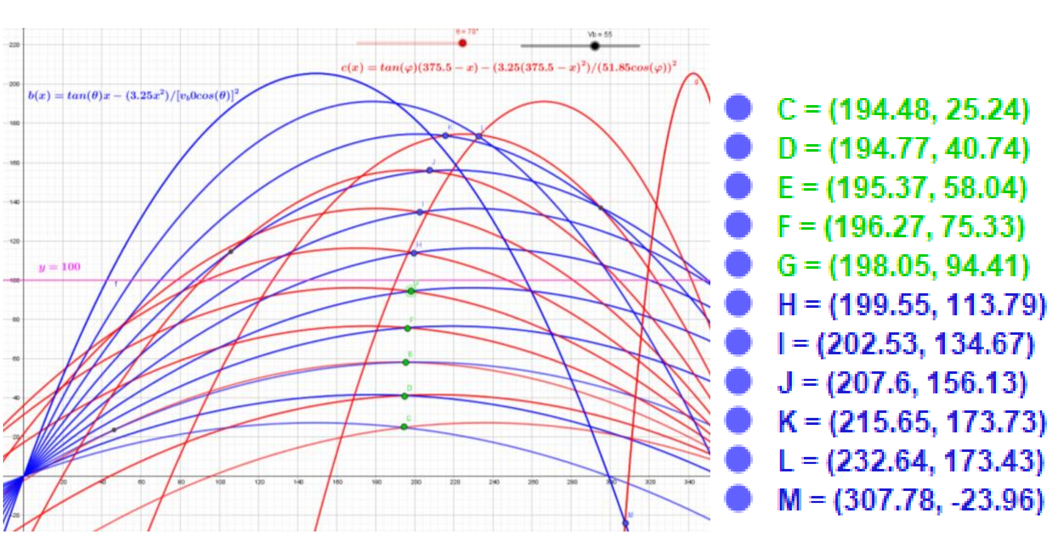
The points at which the car intercepts the ball successfully midair are all highlighted green. Any other points are discarded either because, the ceiling has obstructed the paths of the car and the ball or the points of intersection of the curves are below the x-axis.

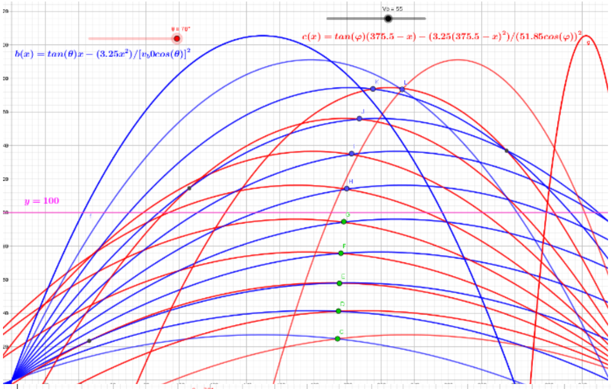



y = 100
b(x) = tan(θ)x − (3.25x²)/[v₀cos(θ)]²
c(x) = tan(φ)(375.5 − x) − (3.25(375.5 − x)²)/(51.85cos(φ))²
C = (148.5, -12.02)
D = (146.09, -0.86)
E = (143.9, 9.87)
F = (140, 20)
G = (137.11, 30.23)
H = (132.08, 39.42)
I = (126.17, 48.18)
J = (119.38, 55.62)
K = (110.19, 61.97)
L = (99.68, 66.13)
M = (86.32, 68.1)

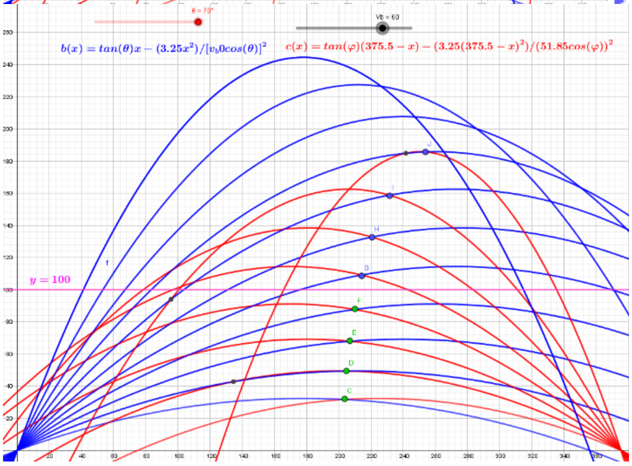

y = 100
b(x) = tan(θ)x − (3.25x²)/[v₀cos(θ)]²
c(x) = tan(φ)(375.5 − x) − (3.25(375.5 − x)²)/(51.85cos(φ))²
C = (172.95, 8.74)
D = (172.34, 22.59)
E = (170.92, 36.23)
F = (169.29, 50.28)
G = (167.25, 64.13)
H = (164.4, 77.57)
I = (160.33, 91.02)
J = (155.64, 104.05)
K = (149.12, 115.25)
L = (140.16, 124.01)
M = (127.54, 127.47)


y = 100
c(x) = tan(φ)(375.5 − x) − (3.25(375.5 − x)²)/(51.85cos(φ))²
b(x) = tan(θ)x − (3.25x²)/[v₀cos(θ)]²
C = (184.04, 17.49)
D = (183.44, 32.39)
E = (183.74, 47.9)
F = (182.85, 63.4)
G = (182.55, 78.91)
H = (181.36, 95.9)
I = (179.57, 112.9)
J = (178.08, 129.3)
K = (175.69, 144.21)
L = (171.22, 153.75)
M = (164.67, 151.63)

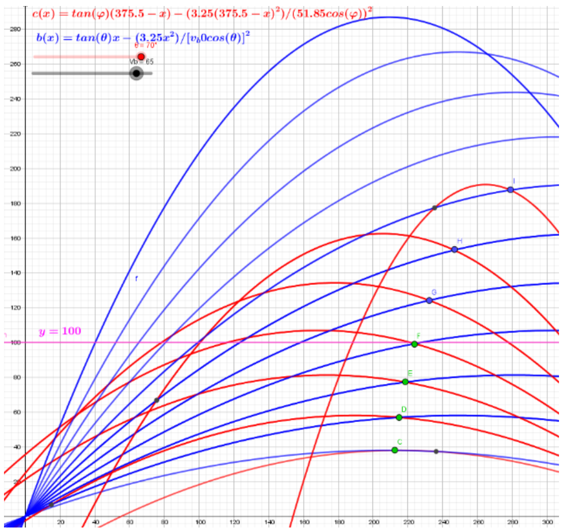

y = 100
b(x) = tan(θ)x − (3.25x²)/[v₀cos(θ)]²
c(x) = tan(φ)(375.5 − x) − (3.25(375.5 − x)²)/(51.85cos(φ))²
C = (194.48, 25.24)
D = (194.77, 40.74)
E = (195.37, 58.04)
F = (196.27, 75.33)
G = (198.05, 94.41)
H = (199.55, 113.79)
I = (202.53, 134.67)
J = (207.6, 156.13)
K = (215.65, 173.73)
L = (232.64, 173.43)
M = (307.78, -23.96)



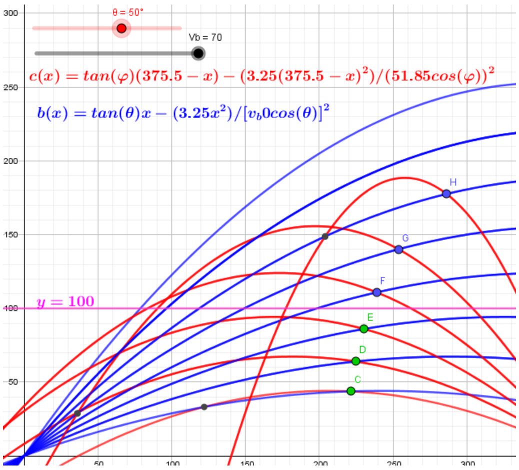
C = (194.21, 24.86)
D = (194.79, 41.38)
E = (195.37, 57.89)
F = (196.24, 75.85)
G = (197.98, 94.68)
H = (199.71, 114.38)
I = (202.61, 135.24)
J = (207.25, 156.1)
K = (215.36, 173.77)
L = (232.74, 173.48)
M = (307.49, -23.81)



C = (203.68, 32.12)
D = (204.74, 49.46)
E = (206.86, 68.21)
F = (210.05, 88.03)
G = (214.3, 108.55)
H = (220.66, 132.61)
I = (231.63, 158.44)
J = (253.92, 185.69)



C = (212.53, 38.14)
D = (215, 56.89)
E = (218.54, 77.41)
G = (232.34, 124.12)
H = (246.85, 153.49)
I = (279.05, 187.81)
F = (223.85, 99)



C = (221.42, 43.96)
D = (224.71, 64.24)
E = (230.19, 86.17)
F = (238.96, 110.83)
G = (253.76, 139.88)
H = (286.09, 177.7)

The graphs above give the points of intersection however the act of individually tabulating each coordinate is tedious. So, I have opted to use the algebraic method of finding (x_i, y_i), where

$$x_i = v_{b0}\cos(\theta)\, t_i, y_i = v_{b0}\sin(\theta)\, t_i - 3.25{t_i}^2, and\ t_i = \frac{375.5}{v_{b0}\cos(\theta) + 51.85\cos(\varphi)}$$

The table uses the excel formula =(375.5)/(M$3*COS($L7*PI()/180)+(51.85*COS(B7*PI()/180))) to find the time of collision.

Time taken for the car to intercept the ball (ti) in seconds									
Angle(θ°)	Velocity of the ball(m/s)								
	30	35	40	45	50	55	60	65	70
20	N/A	N/A	N/A	4.09	3.91	3.75	3.61	3.48	3.36
25	N/A	N/A	4.40	4.22	4.05	3.90	3.77	3.65	3.54
30	N/A	4.75	4.55	4.38	4.23	4.10	3.98	3.88	3.80
35	5.11	4.91	4.74	4.59	4.46	4.36	4.27	4.21	N/A
40	5.28	5.11	4.96	4.84	4.75	4.69	N/A	N/A	N/A
45	5.48	5.34	5.23	5.16	5.12	N/A	N/A	N/A	N/A
50	5.71	5.61	5.56	5.55	N/A	N/A	N/A	N/A	N/A
55	5.97	5.93	5.95	N/A	N/A	N/A	N/A	N/A	N/A
60	6.27	6.30	6.41	N/A	N/A	N/A	N/A	N/A	N/A
65	6.61	6.73	N/A	N/A	N/A	N/A	N/A	N/A	N/A
70	6.98	7.21	N/A	N/A	N/A	N/A	N/A	N/A	N/A

Table 2

From the results above we can notice that a ball with a high angle of projection and a low velocity we have more time to react and intercept the ball compared to a lower angle of projection and a higher velocity. This is important to note because in game we have to make very quick decisions on positioning and ball retrieval.

The excel formulae used for the following tables =C$20*COS($A23*PI()/180)*N6 and =M$20*SIN(L24*PI()/180)*M7-3.25*M7^2

x-coordinates of Intersection									
Angle(θ°)	Velocity of the ball(m/s)								
	30	35	40	45	50	55	60	65	70
20	N/A	N/A	N/A	172.97	183.91	194.07	203.57	212.52	220.99
25	N/A	N/A	159.64	172.03	183.63	194.58	205.00	215.00	224.69
30	N/A	143.86	157.71	170.78	183.26	195.28	207.02	218.61	230.22
35	125.57	140.77	155.22	169.15	182.75	196.25	209.89	223.94	N/A
40	121.35	136.93	152.06	167.01	182.08	197.61	N/A	N/A	N/A
45	116.24	132.17	148.04	164.22	181.16	N/A	N/A	N/A	N/A
50	110.11	126.32	142.95	160.54	N/A	N/A	N/A	N/A	N/A
55	102.78	119.13	136.47	N/A	N/A	N/A	N/A	N/A	N/A
60	94.08	110.32	128.20	N/A	N/A	N/A	N/A	N/A	N/A
65	83.78	99.53	N/A	N/A	N/A	N/A	N/A	N/A	N/A
70	71.64	86.35	N/A	N/A	N/A	N/A	N/A	N/A	N/A

Table 3

y-coordinates of Intersection									
Angle(θ)	Velocity of the ball(m/s)								
	30	35	40	45	50	55	60	65	70
20	N/A	N/A	N/A	8.58	17.14	24.81	31.51	38.01	43.75
25	N/A	N/A	11.54	22.40	32.26	41.21	49.65	56.96	64.01
30	N/A	9.96	23.69	35.98	47.60	48.12	68.05	77.20	86.05
35	3.07	20.30	35.75	49.65	63.26	75.76	88.03	99.32	N/A
40	11.21	30.14	47.56	63.86	79.35	94.32	N/A	N/A	N/A
45	18.65	39.48	59.12	77.66	95.83	N/A	N/A	N/A	N/A
50	25.26	47.81	70.16	91.21	N/A	N/A	N/A	N/A	N/A
55	30.83	55.69	79.91	N/A	N/A	N/A	N/A	N/A	N/A
60	35.11	62.01	88.51	N/A	N/A	N/A	N/A	N/A	N/A
65	37.76	66.48	N/A	N/A	N/A	N/A	N/A	N/A	N/A
70	38.39	67.79	N/A	N/A	N/A	N/A	N/A	N/A	N/A

Table 4

With Table 3 I made an observation that I found quite interesting. Through the range of velocities between $45ms^{-1}$ and $55ms^{-1}$ the difference between consecutive x-coordinates was getting considerably smaller and they were very close to each other. After some thought to this I attributed it to the fact the velocity of the ball was quite close to the velocity of the car.

Modelling the motion of the ball after the collision

Finding resultant velocity and angle of projection at the collision

At the point of collision, the aim is to now bounce the ball at such an angle (α) that it bounces into the goal directly. I need to find the range of angles(α) the ball makes with the horizontal as it is being projected when being hit by the car and its resultant velocity(v_r) after the collision. To find the resultant velocity I will need to find v_{bxi} and v_{byi} which are respectively the horizontal and vertical component of the velocity at the time of collision. Unlike the horizontal component the vertical component changes with time.

It is important to note that for this exploration we are assuming the car to be a stationary wall at the point of collision to help calculate the resultant velocity as the collision will be perfectly elastic.

We know that,

$$V_{byi} = v_{b0}\sin(\varphi) - 6.5t_i$$

For the table I used the formula ==M$35*SIN($L39*PI()/180)-6.5*B39.

Vertical velocity of the ball at point of intersection									
Angle(θ°)	Velocity of the ball(m/s)								
	30	35	40	45	50	55	60	65	70
20	N/A	N/A	N/A	-11.20	-8.34	-5.60	-2.95	-0.38	2.10
25	N/A	N/A	-11.72	-8.40	-5.21	-2.13	0.85	3.75	6.56
30	N/A	-13.35	-9.59	-5.99	-2.51	0.85	4.10	7.26	10.32
35	-16.01	-11.84	-7.85	-4.02	-0.32	3.23	6.66	9.94	N/A
40	-15.04	-10.70	-6.54	-2.57	1.24	4.87	N/A	N/A	N/A
45	-14.41	-9.96	-5.74	-1.73	2.05	N/A	N/A	N/A	N/A
50	-14.13	-9.68	-5.50	-1.60	N/A	N/A	N/A	N/A	N/A
55	-14.25	-9.90	-5.90	N/A	N/A	N/A	N/A	N/A	N/A
60	-14.79	-10.66	-7.02	N/A	N/A	N/A	N/A	N/A	N/A
65	-15.76	-12.02	N/A	N/A	N/A	N/A	N/A	N/A	N/A
70	-17.20	-14.00	N/A	N/A	N/A	N/A	N/A	N/A	N/A

Table 5

Now using Pythagoras theorem I can find the resultant velocity(v_r).

$$v_r = \sqrt{{v_{bxi}}^2 + {v_{byi}}^2};\ where\ v_{bxi} = v_{bx} = v_{b0}\cos(\theta)$$

Resultant velocity of the ball at the collision									
Angle(θ°)	Velocity of the ball(m/s)								
	30	35	40	45	50	55	60	65	70
20	N/A	N/A	N/A	43.74	47.72	51.99	56.46	61.08	65.81
25	N/A	N/A	38.10	41.64	45.61	49.89	54.39	59.03	63.78
30	N/A	33.12	35.94	39.43	43.37	47.64	52.12	56.76	61.49
35	29.33	31.02	33.69	37.08	40.96	45.17	49.60	54.17	N/A
40	27.46	28.87	31.33	34.57	38.32	42.41	N/A	N/A	N/A
45	25.64	26.68	28.86	31.87	35.41	N/A	N/A	N/A	N/A
50	23.91	24.49	26.29	28.97	N/A	N/A	N/A	N/A	N/A
55	22.34	22.38	23.69	N/A	N/A	N/A	N/A	N/A	N/A
60	21.06	20.49	21.20	N/A	N/A	N/A	N/A	N/A	N/A
65	20.23	19.06	N/A	N/A	N/A	N/A	N/A	N/A	N/A
70	20.02	18.42	N/A	N/A	N/A	N/A	N/A	N/A	N/A

Table 6

Since we know that the motion of the ball after the bounce is similar to that of the car in the sense that it is in the same direction and starts at a point different to the origin, we can get the equation of motion of the ball after the collision as:

$$y = (x_i - x)\left(\tan(\alpha) - \frac{3.25(x_i - x)}{(v_r\cos(\alpha))^2}\right) + y_i$$

Let the ball hit the wall at (0, h). Since I know the height of the goal is 24.5m. We have $0m \le h \le 24.5m$

Substituting $x = 0, y = h$ into the above equation:

$$(x_i)\left(\tan(\alpha) - \frac{3.25(x_i)}{(v_r\cos(\alpha))^2}\right) + y_i = h$$

$$(x_i)\left(\tan(\alpha) - \frac{3.25(x_i)\sec^2\alpha}{(v_r)^2}\right) + y_i = h$$

We can use the trigonometric identity $\sec^2\alpha = \tan^2\alpha + 1$ to get the following:

$$\tan(\alpha) - \frac{3.25(x_i)}{{v_r}^2}\tan^2(\alpha) - \frac{3.25(x_i)}{{v_r}^2} = \frac{h - y_i}{x_i}$$

By multiplying through by ${v_r}^2$ we get:

$${v_r}^2\tan(\alpha) - 3.25(x_i)\tan^2(\alpha) - 3.25(x_i) = \frac{(h - y_i)({v_r}^2)}{x_i}$$

$$3.25(x_i)\tan^2(\alpha) - {v_r}^2\tan(\alpha) + 3.25(x_i) + \frac{(h - y_i)({v_r}^2)}{x_i} = 0$$

$$\tan(\alpha) = \frac{{v_r}^2 \pm \sqrt{{v_r}^4 - 4(3.25(x_i))\left[3.25(x_i) + \frac{(h - y_i)({v_r}^2)}{x_i}\right]}}{6.5(x_i)} \longrightarrow \text{Formula 2}$$

The above formula is valid if the discriminant is greater than or equal to 0:

$${v_r}^4 - 4\big(3.25(x_i)\big)\left[3.25(x_i) + \frac{(h - y_i)({v_r}^2)}{x_i}\right] \geq 0$$

$$\frac{{v_r}^4}{4\big(3.25(x_i)\big)} \geq 3.25(x_i) + \frac{(h - y_i)({v_r}^2)}{x_i}$$

$$\frac{{v_r}^4}{4\big(3.25(x_i)\big)} - 3.25(x_i) \geq \frac{(h - y_i)({v_r}^2)}{x_i}$$

$$\left[\frac{{v_r}^4}{4\big(3.25(x_i)\big)} - 3.25(x_i)\right]\frac{x_i}{{v_r}^2} \geq h - y_i$$

$$h \leq \left[\frac{{v_r}^4}{4\big(3.25(x_i)\big)} - 3.25(x_i)\right]\frac{x_i}{{v_r}^2} + y_i$$

With the equation above we can get the maximum height the ball can go when $x = 0$. These heights are shown in the table below using the excel formula =(((B54^4)/(4*3.25*B24))-3.25*B24)*(B24/(B54^2))+M24. It is important to note that h has a range of $0 \leq h \leq 24.5$ so in the table below I made it so any value that exceeds 24.5 will be 24.5 as that is the desired height.

The maximum height the ball can reach when x=0 with limit of 24.5m									
Angle(θ)	Velocity of the ball(m/s)								
	30	35	40	45	50	55	60	65	70
20	N/A	N/A	N/A	24.5	24.5	24.5	24.5	24.5	24.5
25	N/A	N/A	24.5	24.5	24.5	24.5	24.5	24.5	24.5
30	N/A	24.5	24.5	24.5	24.5	24.5	24.5	24.5	24.5
35	9.6	24.5	24.5	24.5	24.5	24.5	24.5	24.5	N/A
40	5.8	21.1	24.5	24.5	24.5	24.5	N/A	N/A	N/A
45	2.4	14.5	24.5	24.5	24.5	N/A	N/A	N/A	N/A
50	0.3	7.5	24.5	24.5	N/A	N/A	N/A	N/A	N/A
55	0.5	2.2	15.2	N/A	N/A	N/A	N/A	N/A	N/A
60	4.4	0.1	4.2	N/A	N/A	N/A	N/A	N/A	N/A
65	13.5	5.8	N/A	N/A	N/A	N/A	N/A	N/A	N/A
70	24.5	22.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A

Table 7

The angle α will be at the minimum if the ball enters the goal at (0, 0). Therefore we can substitute h=0 in Equation 1:

$$\therefore \tan(\alpha) = \frac{{v_r}^2 \pm \sqrt{{v_r}^4 - 4(3.25(x_i))\left[3.25(x_i) + \frac{(-y_i)({v_r}^2)}{x_i}\right]}}{6.5(x_i)}$$

Minimum angle at which the ball should be bounced off the car(α°)									
Angle(θ°)	Velocity of the ball(m/s)								
	30	35	40	45	50	55	60	65	70
20	N/A	N/A	N/A	14.8	10.1	6.2	3.1	0.4	-1.8
25	N/A	N/A	17.9	11.6	6.6	2.4	-1.0	-3.6	-5.9
30	N/A	23.7	15.5	8.8	3.3	1.9	-4.5	-7.3	-9.7
35	33.1	22.4	13.5	6.3	0.5	-4.1	-7.8	-10.6	N/A
40	33.2	21.7	12.1	4.3	-1.9	-6.6	N/A	N/A	N/A
45	34.2	21.9	11.4	3.1	-3.3	N/A	N/A	N/A	N/A
50	36.2	23.5	11.9	3.2	N/A	N/A	N/A	N/A	N/A
55	33.7	26.3	14.4	N/A	N/A	N/A	N/A	N/A	N/A
60	24.9	28.7	19.4	N/A	N/A	N/A	N/A	N/A	N/A
65	14.6	17.0	N/A	N/A	N/A	N/A	N/A	N/A	N/A
70	2.6	2.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A

Table 8

Table 8 shows the minimum angle (α) the ball needs to bounce of the car in order to go directly in the goal. I have used the excel formula =ATAN((B24-SQRT(((B24)^2)-4*((3.25*B24^2)/(B54^2))*(((3.25*B24^2)/(B54^2))-M24)))/(2*((3.25*B24^2)/(B54^2))))*180/PI() in the table.

To find the maximum angle that the ball should be projected in order to enter the goal using the heights in Table 7. I will use the equation:

$$\tan(\alpha) = \frac{{v_r}^2 \pm \sqrt{{v_r}^4 - 4(3.25(x_i))\left[3.25(x_i) + \frac{(h - y_i)({v_r}^2)}{x_i}\right]}}{6.5(x_i)}$$

For the table below I am using the following formula =ATAN(((B54^2)-SQRT((B54^4)-4*(3.25*(B24))*(3.25*(B24)+(((X54-M24)*(B54^2))/(B24)))))/(6.5*B24))*180/PI().

Maximum Angle at which the ball should be bounced off the car(α)									
Angle(θ°)	Velocity of the ball(m/s)								
	30	35	40	45	50	55	60	65	70
20	N/A	N/A	N/A	23.9	18.3	13.8	10.2	7.1	4.6
25	N/A	N/A	28.6	20.9	14.8	10.0	6.0	2.9	0.2
30	N/A	38.7	26.6	18.2	11.5	9.5	2.2	-1.2	-3.9
35	46.5	40.2	25.2	15.8	8.5	3.0	-1.4	-4.8	N/A
40	43.7	43.1	24.5	13.8	6.1	0.2	N/A	N/A	N/A
45	41.0	39.6	25.3	12.9	4.5	N/A	N/A	N/A	N/A
50	38.6	36.0	29.9	13.6	N/A	N/A	N/A	N/A	N/A
55	36.8	32.9	32.3	N/A	N/A	N/A	N/A	N/A	N/A
60	36.0	30.4	28.0	N/A	N/A	N/A	N/A	N/A	N/A
65	36.9	28.9	N/A	N/A	N/A	N/A	N/A	N/A	N/A
70	30.4	31.2	N/A	N/A	N/A	N/A	N/A	N/A	N/A

Table 9

From Tables 8 and 9 I am now able to see the range of angles (α) to bounce the ball off the car so that the ball can go into the goal after a single touch. Interestingly enough, I can see that as the velocity of the ball increases the range of angle that the ball can be bounced gets smaller and smaller. This is mainly due to the fact that the ball and the car are intersecting further away from the goal. This means that as the point of intersection gets further and further away the range of angles decreases.

Ranges of angles the car needs to tilt

Now I need to find the range of angles that the car needs to tilt midair (β) in order to hit the ball directly into the goal. For that I will need to use the range of angles in Tables 8 and 9 for the desired projection and the angle(ω) the ball makes with the horizontal at the time of collision.

At the point of intersection(x_i, y_i):

$$\tan(\omega) = \frac{v_{byi}}{v_{bxi}}$$

$$\tan(\omega) = \frac{v_{bo}\sin(\theta) - 6.5t_i}{v_{b0}\cos(\theta)}$$

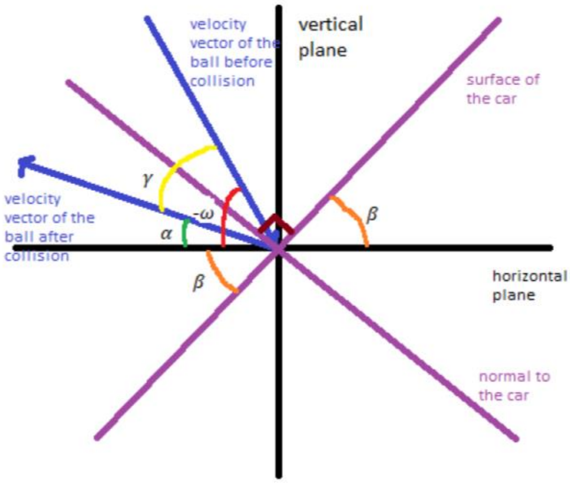
The table below utilizes the above formula to get the angle of collision (ω). It used the following excel formula, =DEGREES(ATAN((AB$35*SIN($W41*PI()/180)-6.5*Q9)/(AB$35*COS($W41*PI()/180)))).

Angle made by the ball at the point of collision(ω°)									
Angle(θ°)	Velocity of the ball(m/s)								
	30	35	40	45	50	55	60	65	70
20	N/A	N/A	N/A	-14.8	-10.1	-6.2	-3.0	-0.4	1.8
25	N/A	N/A	-17.9	-11.6	-6.6	-2.4	0.9	3.6	5.9
30	N/A	-23.8	-15.5	-8.7	-3.3	1.0	4.5	7.3	9.7
35	-33.1	-22.4	-13.5	-6.2	-0.5	4.1	7.7	10.6	N/A
40	-33.2	-21.8	-12.1	-4.3	1.9	6.6	N/A	N/A	N/A
45	-34.2	-21.9	-11.5	-3.1	3.3	N/A	N/A	N/A	N/A
50	-36.2	-23.3	-12.1	-3.2	N/A	N/A	N/A	N/A	N/A
55	-39.6	-26.3	-14.4	N/A	N/A	N/A	N/A	N/A	N/A
60	-44.6	-31.4	-19.4	N/A	N/A	N/A	N/A	N/A	N/A
65	-51.2	-39.1	N/A	N/A	N/A	N/A	N/A	N/A	N/A
70	-59.2	-49.5	N/A	N/A	N/A	N/A	N/A	N/A	N/A

Table 10

Now that we have all the necessary angles I am now able to find the range of angles (β) that the car should tilt in order to bounce the ball back into the goal. From the ω *and* α tables we can see that there are three scenarios to consider when determining the range of angles that the car needs to tilt in order to bounce it directly into the goal. These are:

1. $\omega < 0, \alpha > 0$
2. $\omega > 0, \alpha > 0$
3. $\omega > 0, \alpha < 0$



1. $\omega < 0, \alpha > 0$

The angle β can be found by:

$$\beta + \alpha + \frac{\gamma}{2} = 90°$$

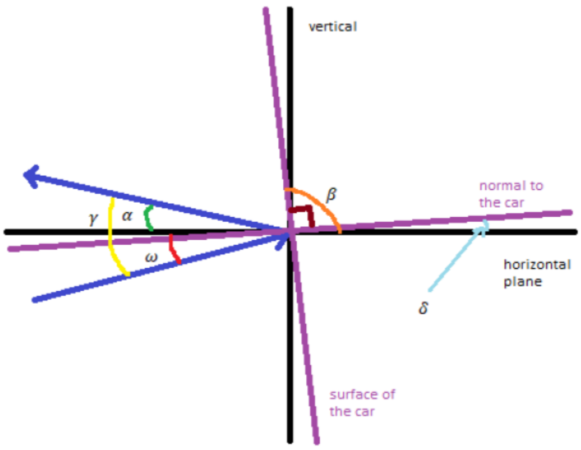
$$\beta = 90° - \left(\frac{\gamma}{2} + \alpha\right)$$

$$\beta = 90° - \left(\frac{-\omega - \alpha}{2} + \alpha\right)$$

$$\beta = 90° - \left(\frac{\alpha - \omega}{2}\right)$$

$$\beta = 90° + \frac{\omega - \alpha}{2}$$

2. $\omega > 0, \alpha > 0$



The angle β can be found by:

$$\beta - \delta = 90° \Longrightarrow \beta = 90° + \delta$$

From the diagram:

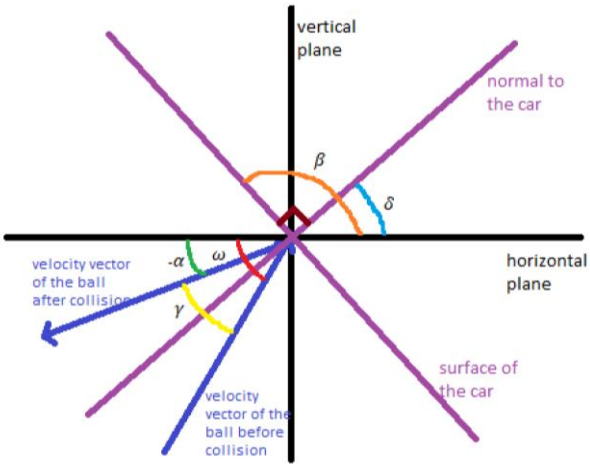
$$\delta = \omega - \frac{\gamma}{2} \; and \; \delta = -\alpha + \frac{\gamma}{2}$$

$$2\delta = \omega - \frac{\gamma}{2} + \left(-\alpha + \frac{\gamma}{2}\right)$$

$$\delta = \frac{\omega - \alpha}{2}$$

$$\therefore \beta = 90° + \frac{\omega - \alpha}{2}$$

3. $\omega > 0, \alpha < 0$



The angle β can be found by:

$$\beta - \delta = 90° \Longrightarrow \beta = 90° + \delta$$

From the diagram:

$$\delta = \omega - \frac{\gamma}{2} \; and \; \delta = -\alpha + \frac{\gamma}{2}$$

$$2\delta = \omega - \frac{\gamma}{2} + \left(-\alpha + \frac{\gamma}{2}\right)$$

$$\delta = \frac{\omega - \alpha}{2}$$

$$\therefore \beta = 90° + \frac{\omega - \alpha}{2}$$

The formula for β is the same for all three scenarios using this formula, the two tables below show the ranges of angles the car needs to tilt (β) in order to project the ball at an angle (α) such that the ball goes in. They both use the formula =90+(X39-M54)/2

Angle at which the car should tilt(β°) to project the ball at the minimum angle(α°)									
Angle(θ°)	Velocity of the ball(m/s)								
	30	35	40	45	50	55	60	65	70
20	N/A	N/A	N/A	75.2	79.9	83.8	87.0	89.6	91.8
25	N/A	N/A	72.1	78.4	83.4	87.6	90.9	93.6	95.9
30	N/A	66.3	74.5	81.2	86.7	89.6	94.5	97.3	99.7
35	56.9	67.6	76.5	83.7	89.5	94.1	97.8	100.6	N/A
40	56.8	68.3	77.9	85.7	91.9	96.6	N/A	N/A	N/A
45	55.8	68.1	78.6	86.9	93.3	N/A	N/A	N/A	N/A
50	53.8	66.6	78.0	86.8	N/A	N/A	N/A	N/A	N/A
55	53.3	63.7	75.6	N/A	N/A	N/A	N/A	N/A	N/A
60	55.2	60.0	70.6	N/A	N/A	N/A	N/A	N/A	N/A
65	57.1	61.9	N/A	N/A	N/A	N/A	N/A	N/A	N/A
70	59.1	64.0	N/A	N/A	N/A	N/A	N/A	N/A	N/A

Table 11.

Angle at which the car should tilt(β°) to project the ball at the maximum angle(α°)									
Angle(θ°)	Velocity of the ball(m/s)								
	30	35	40	45	50	55	60	65	70
20	N/A	N/A	N/A	78.0	80.8	83.1	84.9	86.5	87.7
25	N/A	N/A	75.7	79.6	82.6	85.0	87.0	88.6	89.9
30	N/A	70.7	76.7	80.9	84.2	85.3	88.9	90.6	92.0
35	66.7	69.9	77.4	82.1	85.7	88.5	90.7	92.4	N/A
40	68.1	68.4	77.7	83.1	87.0	89.9	N/A	N/A	N/A
45	69.5	70.2	77.3	83.5	87.8	N/A	N/A	N/A	N/A
50	70.7	72.0	75.1	83.2	N/A	N/A	N/A	N/A	N/A
55	71.6	73.5	73.8	N/A	N/A	N/A	N/A	N/A	N/A
60	72.0	74.8	76.0	N/A	N/A	N/A	N/A	N/A	N/A
65	71.5	75.6	N/A	N/A	N/A	N/A	N/A	N/A	N/A
70	74.8	74.4	N/A	N/A	N/A	N/A	N/A	N/A	N/A

Table 12

The angles have been split into the three scenarios mentioned earlier. The bright yellow cells is for the first scenario $\omega < 0, \alpha > 0$, the bright blue color is for the second scenario $\omega > 0, \alpha > 0$ and the regular green is for the third scenario $\omega > 0, \alpha < 0$. As a player I would prefer the first scenario more than the others because the angles that the car needs to tilt is lower than in the second and third scenarios. This is preferred as there is less time needed to change the angle of the car's tilt which can take too long the more the car needs to tilt.

That means when I'm playing I should always consider at what point in the ball's journey I will collide with the ball. Therefore, it is more favorable to collide when the ball has already reached its maximum height and is going in a downwards direction.

Conclusion

The aim of the exploration was to use the models of the ball's and car's motion in order to find the range of angles that the car needs to tilt in order to hit the ball directly into the goal. From the exploration I can identify that the best conditions for me as player to get the ball is when the ball is projected at a higher angle with a lower velocity. These conditions give me ample time to react and tilt the car in a way that I will successfully score a goal directly. The method of using the equations of motion proved to be especially useful especially when I found small patterns that occurred through the many variations. I can safely say that this exploration was a success as I learnt a lot that can be applied in the game to improve my skill and hopefully let me win more games.

However, there were a few limitations to this exploration because, I only considered the different cases in a single plane when this game occupies a 3D space and at the point of collision I took the car to be a stationary object when it is not. Rocket league is a very expansive game this means that there are many interesting possibilities to explore within the game such as the ball being projected from different positions on the court or how to ricochet the ball from the ceiling into the goal.

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